Necs. factor[1]: $$\boldsymbol{f(n) = \frac{2^n(\pi/2)^{\lfloor n/2 \rfloor}}{n!!} = \frac{2^n \pi^{\lfloor n/2 \rfloor}}{2^{\lfloor n/2 \rfloor} n!!} = \frac{2^{\lceil n/2 \rceil} \pi^{\lfloor n/2 \rfloor}}{n!!}}$$

**volume of an $n$-dimensional sphere**, i.e. an $(n-1)$-sphere: $$V_n = f r^n$$

volume at fraction $k$ of radius: $$V_{n,k} = f(kr)^n = f k^n r^n = k^n V_n$$

if that is half of the total volume: $$V_{n,k} = \frac{V_n}{2} \therefore k^n V_n = \frac{V_n}{2} \therefore k^n = \frac{1}{2} \therefore \boldsymbol{k = \frac{1}{\sqrt[n]{2}}}$$

relative thickness of outer shell containing $V_n/2$: $$\boldsymbol{d = 1 - k = 1 - \frac{1}{\sqrt[n]{2}} = \frac{\sqrt[n]{2}-1}{\sqrt[n]{2}}}$$

| $n$ | $d$ | description |
|---|---|---|
| 1 | 50.0% | **1**-ball = line segment (= degenerate disk, **1D**-*volume* = *length*) |
| **2** | **29.3%** | **2**-ball = **disk** (**2D**-*volume* = *surface area* of disk) |
| **3** | **20.6%** | **3**-ball = **ball** (**3D**-*volume* = daily-life-volume) |
| 4 | 15.9% | **4**-ball (**4D**-*volume* = hypervolume) |
| 5 | 12.9% | etc. |
| 6 | 10.9% | |
| 7 | 9.43% | |
| 8 | 8.30% | |
| 9 | 7.41% | |
| 10 | 6.70% | |

The *more dimensions*, the *thinner* the *outer shell* containing *half* the *volume*, and the *more* practically all *volume* resides *just below* the *surface*!

A **2-D** ***disk*** / **3-D** ***ball*** has **50%** of its entire ***surface area*** / ***volume*** in the ***outermost*** **29.3%** / **20.6%** of its ***radius***!

See also: https://henk-reints.nl/HR-n-spheres.pdf

[1] https://en.wikipedia.org/wiki/Volume_of_an_n-ball#Alternative_forms (search for "without the factor of $\sqrt{2/\pi}$").


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