We have:

Compton wavelength: $\lambda_{\mathrm{C}} = \dfrac{h}{mc}$

I define:

Compton volume: $V_{\mathrm{C}} := \dfrac{4\pi}{3}\left(\dfrac{\lambda_{\mathrm{C}}}{2}\right)^3 = \dfrac{\pi h^3}{6c^3m^3}$

electron:	7.478 972 923 2	$\times 10^{-36}$	m³
neutron:	1.203 153 142 91	$\times 10^{-45}$	m³

Compton density: $\rho_{\mathrm{C}} := \dfrac{m}{V_{\mathrm{C}}} = \dfrac{6c^3m^4}{\pi h^3}$

electron:	9.121 799 928 9	$\times 10^{6}$	kg/m³
neutron:	1.392 114 967 17	$\times 10^{18}$	kg/m³

Compton pressure: $p_{\mathrm{C}} := \dfrac{2}{3}\rho_{\mathrm{C}}c^2 = \dfrac{4c^5m^4}{\pi h^3}$

electron:	7.297 960 82	$\times 10^{21}$	Pa
cf. $p_{\text{degen},e@\rho_{\mathrm{C},e}} = \dfrac{(3\pi^2)^{2/3}\hbar^2}{5m_e}\left(\dfrac{\#e=1}{V_{\mathrm{C},e}}\right)^{5/3} \approx$	8.170 424 68	$\times 10^{20}$	Pa
neutron:	8.341 220 38	$\times 10^{34}$	Pa
cf. ***OBSERVED*** proton pressure[1]:		$\sim 10^{35}$	Pa

Consider a sphere with a core of neutronium $@\rho_{\mathrm{C},n}$ such that it occupies all available internal space, i.e. with deformed neutrons[2] that squeezed out all vacuum between their individual Compton volumes. Suppose this core is surrounded by a shell of plasma consisting of protons & electrons where each pe pair occupies one electron Compton volume (thus neglecting the proton's Compton volume w.r.t. that of the electron), i.e. also with no intermediate empty space. No close-packing, but as compact as possible throughout the thing.

"Existance postulate":

An entity cannot exist unless it is able to fully manifest all of its properties.

Based on common sense, it essentially refines my definition of

exist = being capable of interaction, being observable.

Based on this *existance postulate*, the largest of the Compton volume & the Schwarzschild volume $V_{\mathrm{S}} = 4\pi r_{\mathrm{S}}^3/3$ would be the minimally required volume for a mass in order to be able to exist and it would imply a particle ***cannot*** resist a pressure exceeding its Compton pressure. The above then yields the densest thinkable white dwarf.

We have: $V_{\text{core}} = \dfrac{4\pi}{3}R_{\text{core}}^3$

as well as: $V_{\text{shell}} = \dfrac{4\pi}{3}R_{\text{shell}}^3 - V_{\text{core}} = \dfrac{4\pi}{3}\left(R_{\text{shell}}^3 - R_{\text{core}}^3\right)$

and: $M_{\text{core}} = \rho_{\text{core}}V_{\text{core}} = \dfrac{6c^3m_n^4}{\pi h^3}\cdot\dfrac{4\pi}{3}R_{\text{core}}^3 = \dfrac{8c^3m_n^4}{h^3}R_{\text{core}}^3$

plus: $M_{\text{shell}} = \rho_{\text{shell}}V_{\text{shell}} = \dfrac{8c^3m_e^3m_{pe}}{h^3}\left(R_{\text{shell}}^3 - R_{\text{core}}^3\right)$

Total mass: $M = \dfrac{8c^3}{h^3}\left[m_n^4R_{\text{core}}^3 + m_e^3m_{pe}\left(R_{\text{shell}}^3 - R_{\text{core}}^3\right)\right]$

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[1] Burkert, V.D., Elouadrhiri, L. & Girod, F.X.: "The pressure distribution inside the proton". *Nature* **557,** 396-399 (2018).

[2] Felipe J. Llanes-Estrada, Gaspar Moreno Navarro: "Cubic Neutrons". Modern Physics Letters A 27, 1250033 (2012), arXiv:1108.1859

We define: $R \coloneqq R_{\text{shell}}$

yielding: $M = \frac{8c^3}{h^3}\left[m_n^4 R_{\text{core}}^3 + m_e^3 m_{pe}(R^3 - R_{\text{core}}^3)\right]$

From M_{core}: $R_{\text{core}}^3 = M_{\text{core}} \frac{h^3}{8c^3 m_n^4}$

hence: $M = \frac{8c^3}{h^3}\left[m_n^4 M_{\text{core}} \frac{h^3}{8c^3 m_n^4} + m_e^3 m_{pe}\left(R^3 - M_{\text{core}} \frac{h^3}{8c^3 m_n^4}\right)\right]$

which is: $M = \frac{8c^3}{h^3}\left[m_n^4 M_{\text{core}} \frac{h^3}{8c^3 m_n^4} - m_e^3 m_{pe} M_{\text{core}} \frac{h^3}{8c^3 m_n^4} + m_e^3 m_{pe} R^3\right]$

or: $M = \frac{8c^3}{h^3}\left[(m_n^4 - m_e^3 m_{pe}) M_{\text{core}} \frac{h^3}{8c^3 m_n^4} + m_e^3 m_{pe} R^3\right]$

therefore: $M = M_{\text{core}} \frac{m_n^4 - m_e^3 m_{pe}}{m_n^4} + \frac{8c^3}{h^3} m_e^3 m_{pe} R^3$

which renders: $M_{\text{core}} \frac{m_n^4 - m_e^3 m_{pe}}{m_n^4} = M - \frac{8c^3}{h^3} m_e^3 m_{pe} R^3$

or: $M_{\text{core}} = \frac{m_n^4}{m_n^4 - m_e^3 m_{pe}}\left(M - \frac{8c^3}{h^3} m_e^3 m_{pe} R^3\right)$

which is: $M_{\text{core}} = \frac{m_n^4}{m_n^4 - m_e^3 m_{pe}} M - \frac{m_n^4}{m_n^4 - m_e^3 m_{pe}} m_e^3 m_{pe} \frac{8c^3}{h^3} R^3$

We approximate: $m_{pe} \approx m_n$

yielding: $M_{\text{core}} \approx \frac{m_n^4}{m_n^4 - m_e^3 m_n} M - \frac{m_n^4}{m_n^4 - m_e^3 m_n} m_e^3 m_n \frac{8c^3}{h^3} R^3$

i.e.: $M_{\text{core}} \approx \frac{m_n^3}{m_n^3 - m_e^3} M - \frac{m_n^3}{m_n^3 - m_e^3} m_e^3 m_n \frac{8c^3}{h^3} R^3$

or: $M_{\text{core}} \approx \frac{m_n^3}{m_n^3 - m_e^3}\left(M - \frac{8c^3 m_e^3 m_n}{h^3} R^3\right)$

We calculate: $\frac{m_n^3}{m_n^3 - m_e^3} \approx 1.000\,000\,000\,16 = 1 + 1.6 \times 10^{-10}$

2nd approximation: $M_{\text{core}} \approx M - \frac{8c^3 m_e^3 m_n}{h^3} R^3$

renders: $M_{\text{shell}} \approx \frac{8c^3 m_e^3 m_n}{h^3} R^3$

and: $f_n \coloneqq \frac{M_{\text{core}}}{M} \approx 1 - \frac{8c^3 m_e^3 m_n}{h^3} \frac{R^3}{M}$

which would be the neutronium mass fraction of the thing.

This becomes 0% if: $\frac{8c^3 m_e^3 m_n}{h^3} \frac{R^3}{M} \geq 1$

or: $\frac{3M}{4\pi R^3} \leq \frac{6c^3 m_e^3 m_n}{\pi h^3}$

which equals: $\rho_{\text{C},pe} = \frac{m_{pe}}{V_{\text{C},e}} \approx \frac{6c^3 m_e^3 m_n}{\pi h^3} \approx 2.239\,537\,80 \times 10^8\ \text{kg/m}^3$

Sirius B: $M_{\text{SirB}} \approx 1.018 M_{\odot} \approx 2.024 \times 10^{30}$ kg

$R_{\text{SirB}} \approx 0.0084 R_{\odot} \approx 5850 \times 10^3$ m

$\rho_{\text{SirB}} \approx 2.415 \times 10^9\ \text{kg/m}^3 \approx 10.8\ \rho_{\text{C},pe}$

Based on the aforementioned minimally required volume for a mass in order to be able to exist, Sirius B *must* have a neutronium core, hence it *cannot* consist of only carbon nuclei and electrons.

Shell volume: $V_{\text{shell}} = \frac{M - M_{\text{core}}}{m_{pe} \approx m_n} V_{C,e} = \frac{\pi h^3 M(1-f_n)}{6c^3 m_e^3 m_n}$

core volume: $V_{\text{core}} = \frac{M_{\text{core}}}{m_n} V_{C,n} = \frac{\pi h^3 M f_n}{6c^3 m_n^4}$

total volume: $V = \frac{\pi h^3 M(1-f_n)}{6c^3 m_e^3 m_n} + \frac{\pi h^3 M f_n}{6c^3 m_n^4} = \frac{\pi h^3 M}{6c^3 m_n}\left(\frac{1-f_n}{m_e^3} + \frac{f_n}{m_n^3}\right)$

hence: $M = V\frac{6c^3 m_n}{\pi h^3} / \left(\frac{1-f_n}{m_e^3} + \frac{f_n}{m_n^3}\right) = V\frac{6c^3 m_n}{\pi h^3} / \frac{m_n^3(1-f_n)+m_e^3 f_n}{m_e^3 m_n^3}$

i.e.: $M = V\frac{6c^3 m_n}{\pi h^3} \cdot \frac{m_e^3 m_n^3}{m_n^3(1-f_n)+m_e^3 f_n} = V\frac{\rho_{C,pe} m_n^3}{m_n^3(1-f_n)+m_e^3 f_n}$

hence: $\rho = \frac{\rho_{C,pe} m_n^3}{m_n^3(1-f_n)+m_e^3 f_n} = \frac{\rho_{C,pe} m_n^3}{m_n^3-(m_n^3-m_e^3)f_n}$

approximated by: $\rho \approx \frac{\rho_{C,pe} m_n^3}{m_n^3 - m_n^3 f_n} = \frac{\rho_{C,pe}}{1-f_n}$

yielding: $f_n \approx 1 - \frac{\rho_{C,pe}}{\rho}$

Sirius B: $f_{n,\text{SirB}} = 1 - \frac{\rho_{C,pe}}{\rho_{\text{SirB}}} \approx 1 - \frac{2.23953780\times10^8}{2.415\times10^9} \approx 90.73\%$

We have: $\#n = \frac{M_{\text{core}}}{m_n} = \frac{M f_n}{m_n}$

and: $\#pe = \frac{M_{\text{shell}}}{m_{pe}} \approx \frac{M(1-f_n)}{m_n}$

as well as: $V = \#nV_{C,n} + \#peV_{C,e} = \frac{M f_n}{m_n}\frac{\pi h^3}{6c^3 m_n^3} + \frac{M(1-f_n)}{m_n}\frac{\pi h^3}{6c^3 m_e^3}$

i.e. $V = \frac{4\pi}{3}R^3 = \frac{\pi h^3 M}{6c^3 m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)$

hence: $R = \sqrt[3]{\frac{3}{4\pi} \cdot \frac{\pi h^3 M}{6c^3 m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} = \frac{h}{2c}\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)}$

Newton's law of gravitation: $F = G\frac{Mm}{r^2}$

We will consider a sphere with a core and a surrounding body, i.e. body is outside core.

infinitesimal shell outside core: $dm(r) = 2\pi\rho_{\text{body}} r^2 dr$

We have: $V_{\text{core}} = \frac{4\pi}{3}R_{\text{core}}^3 \therefore M_{\text{core}} = \frac{4\pi}{3}\rho_{\text{core}} R_{\text{core}}^3$

inner body volume within shell: $V_{\text{inbod}}(r) = \frac{4\pi}{3}(r^3 - R_{\text{core}}^3)$

inner body mass: $M_{\text{inbod}}(r) = \rho_{\text{body}} V_{\text{inbod}}(r) = \frac{4\pi}{3}\rho_{\text{body}}(r^3 - R_{\text{core}}^3)$

total M within $r = D \geq R_{\text{core}}$: $M_D = M_{\text{core}} + M_{\text{inbod}}(D)$

Note: D is depth, but measured as distance from centre.

hence: $M_D = \frac{4\pi}{3}\rho_{\text{core}} R_{\text{core}}^3 + \frac{4\pi}{3}\rho_{\text{body}} D^3 - \frac{4\pi}{3}\rho_{\text{body}} R_{\text{core}}^3$

i.e.: $M_D = \frac{4\pi}{3}\rho_{\text{body}}\left(\frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right)$

it gravitates on each outer shell: $dF_D(r) = G\frac{M_D\, dm(r)}{r^2}$

yielding: $dF_D(r) = \frac{4\pi}{3}G\frac{\rho_{\text{body}}\left(\frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right) 4\pi\rho_{\text{body}} r^2 dr}{r^2}$

i.e.: $dF_D(r) = \frac{16\pi^2}{3}G\rho_{\text{body}}^2\left(\frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right) dr$

integration: $$F_D = \frac{16\pi^2}{3} G\rho_{\text{body}}^2 \left(\frac{\rho_{\text{core}} - \rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right) \int_D^R dr$$

yields: $$F_D = \frac{16\pi^2}{3} G\rho_{\text{body}}^2 \left(\frac{\rho_{\text{core}} - \rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right)(R - D)$$

as the total force exerted on all its outer shells by some inner sphere with $r = D$.

the pressure then is: $$P_D = \frac{F_D}{4\pi D^2} = \frac{4\pi}{3} G\rho_{\text{body}}^2 \left(\frac{\rho_{\text{core}} - \rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right) \frac{R - D}{D^2}$$

If no core: $P_D = \frac{4\pi}{3} G\rho^2 D(R - D)$ $\rho = \frac{3M}{4\pi R^3} \Rightarrow$ $P_D = \frac{3GM^2}{4\pi R^6} D(R - D)$

i.e.: $P_D = \frac{4\pi}{3} G\rho^2 (RD - D^2)$ $P_D = \frac{3GM^2}{4\pi R^6} (RD - D^2)$

On www: $P_D = \frac{2\pi}{3} G\rho^2 (R^2 - D^2)$ $P_D = \frac{3GM^2}{8\pi R^6} (R^2 - D^2)$

These do not match. On all web pages & YouTube videos I checked, they **integrate the *pressure***:

$$P_D = \int_{r=D}^{r=R} dP_r = \int_{r=D}^{r=R} \frac{dF_r}{\pi r^2}$$

which is stupidly wrong! When I push you, you feel a pressure equal to the force divided by the surface area of ***my*** hand. And then you push someone else, of course with the surface area of ***your*** hand. In fact, we both push this person, but should we add the *forces* or the *pressures* we exert individually? Suppose some weight on a scale with on top of it a pan of water, 10 cm deep. Another idential scale with an identical weight has a pan with a twice as large bottom surface, also filled to a depth of 10 cm, so it exerts the very same pressure, whilst it contains twice as much water. Adding the pressures of the weight and the pan would yield the very same scale reading, but I firmly insist the 2nd scale reads more than the 1st!

One **should integrated the *force*** exerted by the inner mass up to D on each of its outer shells above D and then divide it by the surface area at D:

$$P_D = \frac{\int_{r=D}^{r=R} dF_D}{4\pi D^2}$$

and I'll stick to *THAT*,

which is: $$P_D = \frac{2\pi G}{3} \rho_{\text{body}}^2 \left(\frac{\rho_{\text{core}} - \rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3\right) \frac{R - D}{D^2}$$

It would yield zero pressure at the very centre of a homogeneous sphere without core (i.e. if $R_{\text{core}} = 0$). **Maximum pressure** would be at a depth of **half the radius**:

$$P_{g,\max} = \left[\frac{2\pi G\rho^2}{3} = \frac{3GM^2}{8\pi R^6}\right] \cdot \left(\frac{1}{2} R\right)^2 = \frac{\pi G\rho^2 R^2}{6} = \frac{\pi G\rho^2}{6} \cdot \sqrt[3]{\frac{4\pi\rho}{3M}}^{\,2} = \mathbf{\frac{3GM^2}{32\pi R^4}}$$

E.g. earth: $P_{g,\max} \approx 43$ GPa **IFF** homogeneous & incompressible!

Would the sphere be *compressible*, its density would increase when diving inwards since the thing would shrink due to this pressure and then the innermost part would have the highest density & pressure, cf. the sun's core.

Note: I now realise that I incorrectly used the gravitational pressure in my document:

http://henk-reints.nl/astro/HR-fall-into-black-hole-slides.pdf

The maximum pressure is merely a fourth of what I there used as the cental pressure and this maximum occurs at a depth of half the radius. The central pressure would equal nought.

if $D = R_{\text{core}}$: $P_{\text{core}} = \frac{2\pi G}{3}\rho_{\text{body}}^2\left(\frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}}R_{\text{core}}^3 + R_{\text{core}}^3\right)\frac{R-R_{\text{core}}}{R_{\text{core}}^2}$

i.e.: $P_{\text{core}} = \frac{2\pi G}{3}\rho_{\text{body}}^2\left(\frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}} + 1\right)R_{\text{core}}(R - R_{\text{core}})$

so: $P_{\text{core}} = \frac{2\pi G}{3}\rho_{\text{body}}^2\left(\frac{\rho_{\text{core}}}{\rho_{\text{body}}}\right)R_{\text{core}}(R - R_{\text{core}})$

hence: $P_{\text{core}} = \frac{2\pi G}{3}\rho_{\text{core}}\rho_{\text{body}}R_{\text{core}}(R - R_{\text{core}})$

yielding: $P_{\text{core}} = \frac{2\pi G}{3}\cdot\rho_{\text{core}}R_{\text{core}}\cdot\rho_{\text{body}}(R - R_{\text{core}})$

But we want to find at what depth the pressure has its maximum.

We have: $P_D = \Gamma(Q^3 + D^3)\frac{R-D}{D^2} = \Gamma Q^3\frac{R-D}{D^2} + \Gamma D(R-D) = \Gamma Q^3\frac{R-D}{D^2} + \Gamma DR - \Gamma D^2$

where: $\Gamma = \frac{2\pi G}{3}\rho_{\text{body}}^2$ and: $Q^3 = \frac{\rho_{\text{core}}-\rho_{\text{body}}}{\rho_{\text{body}}}R_{\text{core}}^3$

Derivative: $\frac{dP_D}{dD} = \Gamma Q^3\frac{D-2R}{D^3} + \Gamma R - 2\Gamma D = \Gamma\left(\frac{Q^3(D-2R)+D^3(R-2D)}{D^3}\right)$

equals zero if: $Q^3(D-2R) + D^3(R-2D) = 0$

i.e.: $Q^3D - 2Q^3R + D^3R - 2D^3D = 0$

hence: $2D^4 - RD^3 - Q^3D + 2Q^3R = 0$

WolframAlpha *does* come up with all four solutions of this quartic equation in D, but it's a terrible behemoth that makes you run away, screaming far above the top of your voice.

Everything has beauty, but not everyone sees it.

— Confucius —

We will presume the core is sufficiently massive to have the maximum pressure at its own surface, as if the pe plasma is like the atmosphere around the earth, where pressure also has its maximum at ground level.

White dwarf values: $\rho_{\text{core}} = \rho_{\text{C},n} = \frac{6c^3m_n^4}{\pi h^3}$

$\rho_{\text{body}} = \rho_{\text{C},pe} = \frac{6c^3m_e^3m_n}{\pi h^3}$

$R_{\text{core}} = \sqrt[3]{M_{\text{core}}\frac{h^3}{8c^3m_n^4}} = \frac{h}{2m_nc}\sqrt[3]{M_{\text{core}}/m_n} = \frac{\lambda_{\text{C},n}}{2}\sqrt[3]{\#n}$

$R_{\text{core}} = \frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n} = \frac{\lambda_{\text{C},n}}{2}\sqrt[3]{Mf_n/m_n}$

yield: $P_{\text{core}} = \frac{2\pi G}{3}\cdot\frac{6c^3m_n^4}{\pi h^3}\frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n}\cdot\frac{6c^3m_e^3m_n}{\pi h^3}\left(R - \frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n}\right)$

i.e.: $P_{\text{core}} = \frac{2\pi G}{3}\cdot\frac{6c^3m_n^4}{\pi h^3}\cdot\frac{6c^3m_e^3m_n}{\pi h^3}\cdot\frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n}\left(R - \frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n}\right)$

hence: $P_{\text{core}} = \frac{12Gc^5m_n^4m_e^3}{\pi h^5}\sqrt[3]{Mf_n/m_n}\left(R - \frac{h}{2m_nc}\sqrt[3]{Mf_n/m_n}\right)$

Found above: $R = \frac{h}{2c}\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)}$

therefore: $P_{\text{core}} = \frac{12Gc^5m_n^4m_e^3}{\pi h^5}\sqrt[3]{\frac{Mf_n}{m_n}}\left(\frac{h}{2c}\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \frac{h}{2m_nc}\sqrt[3]{\frac{Mf_n}{m_n}}\right)$

i.e.: $P_{\text{core}} = \frac{12Gc^5m_n^4m_e^3}{\pi h^5}\sqrt[3]{\frac{Mf_n}{m_n}}\left(\frac{h}{2c}\left[\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{\frac{Mf_n}{m_n^4}}\right]\right)$

hence: $P_{\text{core}} = \frac{12Gc^5m_n^4m_e^3}{\pi h^5}\cdot\frac{h}{2c}\sqrt[3]{\frac{Mf_n}{m_n}}\left(\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{\frac{Mf_n}{m_n^4}}\right)$

which is: $$P_{\text{core}} = \frac{6Gc^4 m_n^4 m_e^3}{\pi h^4} \sqrt[3]{\frac{M f_n}{m_n}} \left(\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{\frac{M f_n}{m_n^4}} \right)$$

from which: $$P_{\text{core}} = \frac{6Gc^4 m_n^2 m_e^3}{\pi h^4} \sqrt[3]{\frac{M f_n m_n^6}{m_n}} \left(\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{\frac{M f_n}{m_n^4}} \right)$$

so: $$P_{\text{core}} = \frac{6Gc^4 m_n^2 m_e^3}{\pi h^4} \sqrt[3]{M f_n m_n^5} \left(\sqrt[3]{\frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{\frac{M f_n}{m_n^4}} \right)$$

i.e.: $$P_{\text{core}} = \frac{6Gc^4 m_n^2 m_e^3}{\pi h^4} \left(\sqrt[3]{M f_n m_n^5 \frac{M}{m_n}\left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{M f_n m_n^5 \frac{M f_n}{m_n^4}} \right)$$

yielding: $$P_{\text{core}} = \frac{6Gc^4 m_n^2 m_e^3}{\pi h^4} \left(\sqrt[3]{M^2 f_n m_n^4 \left(\frac{f_n}{m_n^3} + \frac{1-f_n}{m_e^3}\right)} - \sqrt[3]{M^2 f_n^2 m_n} \right)$$

therefore: $$P_{\text{core}} = \frac{6Gc^4 m_n^2 m_e^3}{\pi h^4} \left(\sqrt[3]{M^2 f_n^2 m_n + M^2 f_n (1-f_n) \frac{m_n^4}{m_e^3}} - \sqrt[3]{M^2 f_n^2 m_n} \right)$$

which would be the pressure at the core's surface.

We can make the pressure: $$P_D = \frac{2\pi G}{3} \rho_{\text{body}}^2 \left(\frac{\rho_{\text{core}} - \rho_{\text{body}}}{\rho_{\text{body}}} R_{\text{core}}^3 + D^3 \right) \frac{R-D}{D^2}$$

dimensionless by either: $P_D^* = P_D / p_{\text{C},e}$ where: $p_{\text{C},e} = \frac{4c^5 m_e^4}{\pi h^3}$

or: $P_D^\# = P_D / p_{\text{C},pe}$ where: $p_{\text{C},pe} = \frac{2}{3} \rho_{\text{C},pe} c^2 \approx \frac{4c^5 m_e^3 m_n}{\pi h^3}$

Note: $\rho_{\text{C},pe}$ would actually be: $\frac{m_p + m_e}{V_{\text{C},p} + V_{\text{C},e}} \approx \frac{m_n}{V_{\text{C},e}}$

For the plasma, it should be impossible to exceed either one of these critical pressures (I see no objective criterion to make a choice between them), i.e. it should be Titanic[3] that $P_D^* > 1$ or $P_D^\# > 1$. The innermost plasma layer would collapse to neutronium.

MAYBE the proton's mass (energy) density contributes to the maximum pressure the pe pair could withstand?

We define:

- height above core's surface: $H = \frac{D - P_{\text{core}}}{R - P_{\text{core}}}$
- "electric Compton force": $F_{e,\lambda_\text{C}} = k_e \frac{e^2}{\lambda_\text{C}^2}$
- grav. force by core on proton: $F_{g,p} = G \frac{M_{\text{core}} m_p}{D^2}$

For Sirius B, a numerical analysis yields:

$H \geq 0.4291$	$P_{\text{core}}^* < 1$	$P_{\text{core}}^\# < 1$	
$0.0120 \leq H < 0.4291$	$P_{\text{core}}^* > 1$	$P_{\text{core}}^\# < 1$	
$H < 0.0120$	$P_{\text{core}}^* > 1$	$P_{\text{core}}^\# > 1$	
$H = 0$		$P_{\text{core}}^\# \approx 129$	$\frac{F_{g,p}}{F_{e,\lambda_\text{C}}} \approx 1.13 \times 10^{-8}$

Within the lowest 1.2%, the plasma pressure would be unbaringly high, so pe pairs would collapse to neutrons. But the electrical force could easily keep the plasma together against gravity. Could there be some sort of "hollow shell" between the core and the plasma? Where plasma is unable to exist? A gas of neutrons, orbiting the core, provided

[3] *Unthinkable* ship?

there is sufficient angular momentum available? Could their decay be in equilibrium with the *pe* pair collapses? Or — what I consider more plausible — would the neutronium core just be less dense?

Due to the *fact* that Sirius B *does* exist with its observed mass and radius, I now *do* see an objective criterion to choose between the two critical pressures.

It must be: $P_D^{\#} = P_D / p_{\mathrm{C},pe} < 1$

But a WD would contain mainly carbon nuclei, so maybe I should not have used $p_{\mathrm{C},pe}$ but: $p_{\mathrm{C},x} = \frac{2}{3} \sum m_i c^2 / \sum V_{\mathrm{C},i}$
as the maximum pressure an entity consisting of particles can withstand.
We find: $p_{\mathrm{C},pe} \approx 1.3407 \times 10^{25}$ Pa
and: $p_{\mathrm{C},{}^{12}_{6}\mathrm{C}} \approx 1.3433 \times 10^{25}$ Pa
which is not a significant difference, so for the sake of simplicity,

I'll stick to: $P_D^{\#} = P_D / p_{\mathrm{C},pe} < 1$

Tipping point: $$P_D = \frac{2\pi G}{3} \rho_{\mathrm{body}}^2 \left(\frac{\rho_{\mathrm{core}} - \rho_{\mathrm{body}}}{\rho_{\mathrm{body}}} R_{\mathrm{core}}^3 + D^3 \right) \frac{R-D}{D^2} = p_{\mathrm{C},pe} = \frac{4c^5 m_e^3 m_n}{\pi h^3}$$

Found before: $$R_{\mathrm{core}}^3 = M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4}$$

hence: $$\frac{2\pi G}{3} \cdot \frac{\pi h^3}{4c^5 m_e^3 m_n} \rho_{\mathrm{body}}^2 \left(\frac{\rho_{\mathrm{core}} - \rho_{\mathrm{body}}}{\rho_{\mathrm{body}}} M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} + D^3 \right) \frac{R-D}{D^2} = 1$$

i.e.: $$\frac{\pi^2 G h^3}{6c^5 m_e^3 m_n} \rho_{\mathrm{body}}^2 \left(\frac{\rho_{\mathrm{core}} - \rho_{\mathrm{body}}}{\rho_{\mathrm{body}}} M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} + D^3 \right) \frac{R-D}{D^2} = 1$$

We also have: $$\rho_{\mathrm{body}} = \rho_{\mathrm{C},pe} = \frac{6c^3 m_e^3 m_n}{\pi h^3}$$

hence: $$\rho_{\mathrm{body}}^2 = \frac{36c^6 m_e^6 m_n^2}{\pi^2 h^6}$$

as well as: $$\rho_{\mathrm{core}} = \rho_{\mathrm{C},n} = \frac{6c^3 m_n^4}{\pi h^3}$$

yielding: $$\frac{\rho_{\mathrm{core}} - \rho_{\mathrm{body}}}{\rho_{\mathrm{body}}} = \frac{\frac{6c^3 m_n^4}{\pi h^3} - \frac{6c^3 m_e^3 m_n}{\pi h^3}}{\frac{6c^3 m_e^3 m_n}{\pi h^3}} = \frac{m_n^3 - m_e^3}{m_e^3}$$

which renders: $$\frac{\pi^2 G h^3}{6c^5 m_e^3 m_n} \cdot \frac{36c^6 m_e^6 m_n^2}{\pi^2 h^6} \left(\frac{m_n^3 - m_e^3}{m_e^3} M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} + D^3 \right) \frac{R-D}{D^2} = 1$$

i.e.: $$\frac{6Gc m_e^3 m_n}{h^3} \left(\frac{m_n^3 - m_e^3}{m_e^3} M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} + D^3 \right) \frac{R-D}{D^2} = 1$$

For simplicity, we approximate: $D = R_{\mathrm{core}}$ at the tipping point, thus ignoring the 1.2% found above for Sirius B.

It yields: $$\frac{6Gc m_e^3 m_n}{h^3} \left(\frac{m_n^3 - m_e^3}{m_e^3} M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} + M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} \right) \frac{R - R_{\mathrm{core}}}{R_{\mathrm{core}}^2} = 1$$

hence: $$\frac{6Gc m_e^3 m_n}{h^3} \left(\frac{m_n^3 - m_e^3}{m_e^3} + 1 \right) M_{\mathrm{core}} \frac{h^3}{8c^3 m_n^4} = \frac{R_{\mathrm{core}}^2}{R - R_{\mathrm{core}}}$$

i.e.: $$\frac{6Gc m_e^3 m_n}{h^3} \cdot \frac{m_n^3}{m_e^3} \cdot \frac{h^3}{8c^3 m_n^4} M_{\mathrm{core}} = \frac{R_{\mathrm{core}}}{R - R_{\mathrm{core}}^2}$$

so: $$\frac{3G}{4c^2} M_{\mathrm{core}} = \frac{R_{\mathrm{core}}^2}{R - R_{\mathrm{core}}}$$

which renders: $$R = \frac{4c^2 R_{\mathrm{core}}^2}{3GM_{\mathrm{core}}} + R_{\mathrm{core}} = \left(1 + \frac{4c^2 R_{\mathrm{core}}}{3GM_{\mathrm{core}}} \right) R_{\mathrm{core}}$$

Found earlier: $M = M_{\text{core}} \frac{m_n^4 - m_e^3 m_{pe}}{m_n^4} + \frac{8c^3}{h^3} m_e^3 m_{pe} R^3$

i.e.: $M = \frac{m_n^4 - m_e^3 m_n}{m_n^4} M_{\text{core}} + \frac{8c^3}{h^3} m_e^3 m_n R^3$

or: $M = \frac{m_n^3 - m_e^3}{m_n^3} M_{\text{core}} + \frac{8c^3}{h^3} m_e^3 m_n R^3 \approx M_{\text{core}} + \frac{8c^3 m_e^3 m_n}{h^3} R^3$

so: $M_{\text{core}} = M - m_n \frac{8c^3 m_e^3}{h^3} R^3 = M - m_n \left(\frac{2R}{\lambda_{\text{C},e}}\right)^3$

we also have: $M_{\text{core}} = \frac{4\pi}{3} R_{\text{core}}^3 \rho_{\text{C},n} = \frac{4\pi}{3} R_{\text{core}}^3 \frac{6c^3 m_n^4}{\pi h^3} = \frac{8c^3 m_n^4}{h^3} R_{\text{core}}^3 = m_n \left(\frac{2R_{\text{core}}}{\lambda_{\text{C},n}}\right)^3$

hence: $R_{\text{core}}^3 = \left(\frac{\lambda_{\text{C},n}}{2}\right)^3 \cdot \frac{M_{\text{core}}}{m_n}$

so: $R_{\text{core}} = \frac{\lambda_{\text{C},n}}{2} \cdot \sqrt[3]{\frac{M_{\text{core}}}{m_n}}$

rendering: $R = \left(1 + \frac{4c^2 \frac{\lambda_{\text{C},n}}{2} \cdot \sqrt[3]{M_{\text{core}}/m_n}}{3GM_{\text{core}}}\right) \frac{\lambda_{\text{C},n}}{2} \cdot \sqrt[3]{M_{\text{core}}/m_n}$

hence: $R = \frac{\lambda_{\text{C},n}}{2} \left(1 + \frac{4c^2}{3G} \cdot \frac{\lambda_{\text{C},n}}{2} \cdot \frac{\sqrt[3]{\frac{M - m_n (2R/\lambda_{\text{C},e})^3}{m_n}}}{M - m_n (2R/\lambda_{\text{C},e})^3}\right) \sqrt[3]{\frac{M - m_n (2R/\lambda_{\text{C},e})^3}{m_n}}$

or: $1 = \frac{\lambda_{\text{C},n}}{2R} \left(1 + \frac{4c^2}{3G} \cdot \frac{\lambda_{\text{C},n}}{2} \cdot \frac{\sqrt[3]{\frac{M - m_n (2R/\lambda_{\text{C},e})^3}{m_n}}}{M - m_n (2R/\lambda_{\text{C},e})^3}\right) \sqrt[3]{\frac{M - m_n (2R/\lambda_{\text{C},e})^3}{m_n}}$

which would essentially relate the approximated minimal R for a given M in order to be able to exist. It would/could consist of a neutronium core surrounded by a pe plasma, each at their Compton density, where the plasma would resist the gravitational pressure.

BUT a numerical analysis yields no solutions ☹.
Equating $D = R_{\text{core}}$ probably is not a realistic approximation.

Please note that all of the above only takes Compton volumes, densities and pressures into account, as well as Newtonian gravitational pressure. It does not do anything relativistic or quantum mechanical. And although I initially had a slight unfounded hope for it, the Chandrasekhar limit of $\sim 1.44 M_\odot$ (which I do not doubt, although — to my knowledge — hardly any observational evidence exists) does of course not jump out of the numerical analysis.

I ignore any crushing of the neutronium core, which I *THINK* is impossible. At the neutron Compton density, a black hole would be ~ 3.64 solar masses and its internal Newtonian gravitational pressure would be way below the neutron Compton pressure. I am convinced smaller black holes cannot & do not exist, simply because greater densities cannot or at least do not exist. To my knowledge, such a density has never ever been *observed*.

I firmly and on beforehand reject each and every *presumption* as a premise for whatever theory.

One should not come up with a theory,
but with an irrefutable deduction from certainties.

Ex falso sequitur quod libet.

From falsehood follows whatever you like.

Ex fabricationibus sequitur stultorum paradisum.

From fabrications follows the fool's paradise.

NL: *Uit verzinselen volgen luchtkastelen.*

According to the Schwarzschild interior solution, the inside of a BH would have a homogeneous *expansive* pressure of:

$$P = \rho c^2 = \frac{3Mc^2}{4\pi r_S^3} = \frac{3Mc^2}{4\pi \frac{8G^3M^3}{c^6}} = \frac{3c^8}{32\pi G^3 M^2}$$

The more mass, the larger the BH becomes and the lower its pressure.

Neutrons will not be crushed, certainly not by a