## See also: $\quad$ https://www.youtube.com/watch?v=pTn6Ewhb27k (Derek Muller's Veritasium) https://www.youtube.com/watch?v=ff0aofh6urU http://henk-reints.nl/astro/HR-speed-of-light.pdf

Measuring the speed of light by sending it to a mirror @d that reflects it back, how Einstein described it in "Zur Elektrodynamik bewegter Körper" (1905).

and: $\quad \eta c=\quad$ returning speed of light.
Einstein: $\quad c=\frac{2 d}{\Delta t}$
(Zur Elektrodynamik bewegter Körper)
hence: $\quad \Delta t=\frac{2 d}{c}$
Mirror's velocity: $\quad v, \beta=\frac{v}{c} \quad\left(v=\frac{\partial d}{\partial t}\right.$ is positive if distance increases)
Way out: $\quad \Delta t_{0}=\frac{d}{\zeta c}$
way back:
$\Delta t_{1}=\frac{d}{\eta c}$
Total:
$\Delta t=\Delta t_{0}+\Delta t_{1}=\frac{d}{\zeta c}+\frac{d}{\eta c}=\frac{\eta c d}{\eta c \zeta c}+\frac{\zeta c d}{\eta c \zeta c}=\frac{\eta c d+\zeta c d}{\eta c \zeta c}=\frac{(\eta+\zeta) d}{\eta \zeta c}$
Obviously: $\quad \frac{2 d}{c}=\frac{(\eta+\zeta) d}{\eta \zeta c} \therefore \frac{(\eta+\zeta)}{\eta \zeta}=2 \therefore \eta+\zeta=2 \eta \zeta$
$\therefore \zeta=2 \eta \zeta-\eta=\eta(2 \zeta-1)$
yielding:
$\eta=\frac{\zeta}{2 \zeta-1}$
(requirement to render $c$ as the
back \& forth average speed of light).
eta and beta as function of zeta ( x in the graph, hor. axis):


Please note: negative zeta is malarkey.
(graph made by Google).
For the curve of $\beta, \zeta$ is presumed a moving object's Doppler factor.

Doesn't the blueshift part $(0<\zeta<1)$ look weird, especially the asymptote at $\zeta=\frac{1}{2}$ ? And wouldn't a negative $\eta$ be rather senseless? Wouldn't this already indicate the whole concept of two speeds of light is bunkum? We have not yet made use of $\zeta$ being equal to the mirror's Doppler factor (only for graphing the $\beta$ curve).

For any meaningful measurement, one should always use non-moving stationary equipment that is pleonastically standing still at one point in space without displacement, i.e. having: $\beta=0 \therefore \zeta=\sqrt{\frac{1+\beta}{1-\beta}}=1$, yielding: $\eta=\frac{\zeta}{2 \zeta-1}=1$, so there is no net Doppler effect and with $\zeta=\eta=1$ we have one single speed of light in both directions. Now we did use $\zeta=$ mirror's Doppler factor, as aforementioned.

Presuming $v \ll c$, we can use the Classical Doppler effect in good approximation. We'll have to multiply the Doppler factors that apply to each one-way journey.

Way out, mirror "observes":
way home, mirror acts like source, we observe it:
Their product equals: $\quad \frac{v_{\mathrm{obs}}}{v_{\mathrm{src}}}=\frac{\zeta-\beta}{\zeta} \cdot \frac{\eta}{\eta+\beta}=\frac{\eta(\zeta-\beta)}{\zeta(\eta+\beta)}$
Total Doppler factor: $\quad \xi:=\frac{v_{\text {src }}}{v_{\mathrm{obs}}}=\frac{\zeta(\eta+\beta)}{\eta(\zeta-\beta)} \quad$ (the reciprocal thereof)
we had found: $\quad \eta=\frac{\zeta}{2 \zeta-1}$
yielding: $\quad \xi=\frac{\zeta\left(\frac{\zeta}{2 \zeta-1}+\beta\right)}{\frac{\zeta}{2 \zeta-1}(\zeta-\beta)}=\frac{\left(\frac{\zeta}{2 \zeta-1}+\beta\right)}{\frac{\zeta-\beta}{2 \zeta-1}}=\frac{(2 \zeta-1)\left(\frac{\zeta}{2 \zeta-1}+\beta\right)}{\zeta-\beta}=\frac{\zeta+\beta(2 \zeta-1)}{\zeta-\beta}=\frac{\zeta-\beta+2 \zeta \beta}{\zeta-\beta}$
WolframAlpha:
Taylor $@ \beta=0: \quad \xi=1+2 \beta+\frac{2}{\zeta} \beta^{2}+\frac{2}{\zeta^{2}} \beta^{3}+\frac{2}{\zeta^{3}} \beta^{4}+\mathcal{O}\left(\beta^{5}\right)$
Classical Doppler effect with just one speed of light:

$$
\begin{equation*}
\frac{c+v}{c} \cdot \frac{c}{c-v}=\frac{1+\beta}{1-\beta}=1+2 \beta+2 \beta^{2}+2 \beta^{3}+2 \beta^{4}+\mathcal{O}\left(\beta^{5}\right) \tag{B}
\end{equation*}
$$

Relativistic:
light source's mirror image seems to distantiate itself
at a velocity of: $\frac{2 \beta}{1+\beta^{2}} \quad$ (relativistic velocity addition)
rel. Doppler: $\quad \sqrt{\frac{1+\frac{2 \beta}{1+\beta^{2}}}{1-\frac{2 \beta}{1+\beta^{2}}}}=\sqrt{\frac{1+\beta^{2}+2 \beta}{1+\beta^{2}-2 \beta}}=\sqrt{\frac{(1+\beta)^{2}}{(1-\beta)^{2}}}=\frac{1+\beta}{1-\beta} \quad$ same as classical!
Obviously, either $[A]$ or $[B]$ is correct, or both are equal, implying $\zeta=1$. Correctness of [A] would still yield just one and only one correct value of $\zeta$, or would $\zeta$ depend on $\beta$ ? And how plausible would $\zeta \neq 1$ be when looking at these equations?
Would measurements confirm correctness of $[B]$, then we're there: $\zeta=\mathbf{1} \therefore \boldsymbol{\eta}=\mathbf{1}$. I do not know if for example radar guns for speed measurements (as used by the police in many countries) are accurate up to the $2^{\text {nd }}$ order. Fact is that there can be just one $\{\zeta, \eta\}$ pair matching reality and to me, $\{1,1\}$ seems the most plausible.

In what I can find about Doppler measurements of satellites, I see no anomalies. Especially GPS is very accurate! See http://henk-reints.nl/astro/HR-speed-of-light.pdf for a bit more about GPS, which effectively does measure the one-way speed of light in all directions and it does have an accuracy within 8 metres (in $95 \%$ of open-field measurements)!

As mentioned above, it should also be that:
yielding:

$$
\begin{aligned}
& \zeta=\sqrt{\frac{1+\beta}{1-\beta}} \therefore \beta=\frac{\zeta^{2}-1}{\zeta^{2}+1} \quad \text { (which is relativistic, hence exact) } \\
& \frac{v_{\text {src }}}{v_{\text {obs }}}=\frac{\zeta+(2 \zeta-1) \beta}{\zeta-(2 \zeta-1) \beta}=\frac{\zeta+(2 \zeta-1) \frac{\zeta^{2}-1}{\zeta^{2}+1}}{\zeta-(2 \zeta-1) \frac{\zeta^{2}-1}{\zeta^{2}+1}}=\frac{\zeta\left(\zeta^{2}+1\right)+(2 \zeta-1)\left(\zeta^{2}-1\right)}{\zeta\left(\zeta^{2}+1\right)-(2 \zeta-1)\left(\zeta^{2}-1\right)} \\
& =\frac{\left(\zeta^{3}+\zeta\right)+\left(2 \zeta^{3}-\zeta^{2}-2 \zeta+1\right)}{\left(\zeta^{3}+\zeta\right)-\left(2 \zeta^{3}-\zeta^{2}-2 \zeta+1\right)}=\frac{\zeta^{3}+\zeta+2 \zeta^{3}-\zeta^{2}-2 \zeta+1}{\zeta^{3}+\zeta-2 \zeta^{3}+\zeta^{2}+2 \zeta-1}=\frac{3 \zeta^{3}-\zeta^{2}-\zeta+1}{-\zeta^{3}+\zeta^{2}+3 \zeta-1}
\end{aligned}
$$

It should also be that: $\quad \frac{v_{\mathrm{src}}}{v_{\mathrm{obs}}}=\zeta \quad$ (actually the premise of the current derivation).
Difference would be:

$$
\frac{v_{\mathrm{src}}}{v_{\mathrm{obs}}}-\zeta=\frac{3 \zeta^{3}-\zeta^{2}-\zeta+1}{-\zeta^{3}+\zeta^{2}+3 \zeta-1}-\zeta=\frac{(1-\zeta)\left(\zeta^{3}+3 \zeta^{2}-\zeta-1\right)}{\zeta^{3}-\zeta^{2}-3 \zeta+1}
$$




WolframAlpha:
Taylor @ $\zeta=1$ :
$(\zeta-1)+3(\zeta-1)^{2}+(\zeta-1)^{3}+3(\zeta-1)^{4}+\mathcal{O}\left((\zeta-1)^{5}\right)$
with $z=\zeta-1$ :
$z+3 z^{2}+z^{3}+3 z^{4}+\mathcal{O}\left(z^{5}\right)$
In $1^{\text {st }}$ order, the difference equals $z$, yielding a factor of 2 w.r.t. Doppler effect already being $z$ !

Roots:

$$
\zeta \in\{1, \sim 0.67513, \sim-0.46081, \sim-3.2143\}
$$

Please remember the usage of the classical Doppler effect, which only applies to $\zeta \approx 1$. Together with the weirdness of the graph, $\zeta \ll 1$ or $\zeta \gg 1$ makes no sense at all and $\zeta<0$ is malarkey. This leaves $\zeta=1$, meaning that the mirror is not moving, i.e. $v=$ 0 . And it yields $\eta=1$ and then the speed of light definitely is the same in both directions.

## YOUR brainchildren are wrong, not the cosmos!

(My intention was to use brainchild as a pejorative term!)

