Einstein's velocity addition

("Zur Elektrodynamik bewegter Körper", Ann. Phys. 17 (1905), 891-921):

@ p. 906:
$$U = \frac{\sqrt{v^2+w^2+2vw\cos\alpha - \left(\frac{vw\sin\alpha}{c}\right)^2}}{1+\frac{vw\cos\alpha}{c^2}}$$

dimensionless:
$$\frac{U}{c} = \frac{\sqrt{\frac{v^2}{c^2}+\frac{w^2}{c^2}+2\frac{v}{c}\cdot\frac{w}{c}\cos\alpha - \left(\frac{v}{c}\cdot\frac{w}{c}\sin\alpha\right)^2}}{1+\frac{v}{c}\cdot\frac{w}{c}\cos\alpha}$$

which is:
$$\beta_0 = \frac{\sqrt{\beta_1^2+\beta_2^2+2\beta_1\beta_2\cos\alpha - \beta_1^2\beta_2^2\sin^2\alpha}}{1+\beta_1\beta_2\cos\alpha}$$

Note: $\{\alpha, \beta_0, \beta_1\}$ observed in stationary frame,
β_2 in frame moving at β_1 in stat. frame.

Parallel: $(\alpha = 0) \Rightarrow \quad \boldsymbol{\beta_0} = \frac{\sqrt{\beta_1^2+\beta_2^2+2\beta_1\beta_2}}{1+\beta_1\beta_2} = \frac{\boldsymbol{\beta_1+\beta_2}}{\boldsymbol{1+\beta_1\beta_2}}$

Perpendicular: $\left(\alpha = \frac{\pi}{2}\right) \Rightarrow \quad \boldsymbol{\beta_0} = \sqrt{\boldsymbol{\beta_1^2 + \beta_2^2 - \beta_1^2\beta_2^2}}$

Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \therefore \gamma^2(1-\beta^2) = 1 \therefore \gamma^2 - \gamma^2\beta^2 = 1 \therefore \gamma^2 - 1 = \gamma^2\beta^2$$

$$\therefore \boldsymbol{\beta^2 = \frac{\gamma^2-1}{\gamma^2} = 1 - \frac{1}{\gamma^2}}$$

Perpendicular velocity addition:

$$\beta_0 = \sqrt{\beta_1^2 + \beta_2^2 - \beta_1^2\beta_2^2}$$

$$\boldsymbol{\gamma_0 =} \frac{1}{\sqrt{1-\beta_0^2}} = \frac{1}{\sqrt{1-\beta_1^2-\beta_2^2+\beta_1^2\beta_2^2}} = \frac{1}{\sqrt{1 - \left(1-\frac{1}{\gamma_1^2}\right) - \left(1-\frac{1}{\gamma_2^2}\right) + \left(1-\frac{1}{\gamma_1^2}\right)\left(1-\frac{1}{\gamma_2^2}\right)}}$$

$$= \frac{1}{\sqrt{1 \quad -1+\frac{1}{\gamma_1^2} \quad -1+\frac{1}{\gamma_2^2} \quad + \left(1-\frac{1}{\gamma_1^2}-\frac{1}{\gamma_2^2}+\frac{1}{\gamma_1^2}\cdot\frac{1}{\gamma_2^2}\right)}} = \frac{1}{\sqrt{\frac{1}{\gamma_1^2\gamma_2^2}}} \boldsymbol{= \gamma_1\gamma_2}$$

We have: $\boldsymbol{E_{\text{tot}} = \gamma m c^2}$

hence: $\boldsymbol{\gamma = E_{\text{tot}}/mc^2}$

which physically is: dimensionless specific energy (i.e. energy per mass).

For perpendicular motions,
the total specific energy is the
product of the individual values!

$$\boldsymbol{\gamma_0 = \gamma_1 \gamma_2} = \frac{E_{\text{tot},1} E_{\text{tot},2}}{m_1 m_2 c^4} = \frac{(E_{\text{kin},1} + m_1 c^2)(E_{\text{kin},2} + m_2 c^2)}{m_1 m_2 c^4}$$

Parallel: $$\beta_0 = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2}$$

perpendicular: $$\beta_0 = \sqrt{\beta_1^2 + \beta_2^2 - \beta_1^2\beta_2^2} \quad \& \quad \gamma_0 = \gamma_1\gamma_2$$

suffix 0: object's velocity w.r.t. stationary observer;
suffix 2: object's velocity w.r.t. some moving ref. point P;
suffix 1: P's velocity w.r.t. stationary observer
(consistent with the aforementioned, but 1 & 2 are of course interchangeable).

$$(\beta \to 0) \Leftrightarrow (\gamma \to 1) \qquad (\beta \to 1) \Leftrightarrow (\gamma \to \infty)$$

If $\{\beta_1, \gamma_1\} \to \{1, \infty\}$ **The more an object's velocity w.r.t. P approaches c,**
then $\{\beta_0, \gamma_0\} \to \{1, \infty\}$ **the more its velocity w.r.t. stat. obs. approaches c,**
totally indep. of $\{\beta_2, \gamma_2\}$ **i.e. the more its velocity becomes absolute, namely c.**

Relativity yields absoluteness...

Lorentz contraction of a moving object:

$L :=$ length of moving object, measured in direction of motion;

$$\boldsymbol{L_{\mathbf{seenByStationary}} = L_{\mathbf{seenByComoving}}\sqrt{1-\beta^2}}$$

If *you* are speeding, then from *your* perspective the street is moving, hence contracted by the same factor $\sqrt{1-\beta^2}$.

The faster you go, the shorter the street appears to you
(cf. light experiencing zero time and distance).

Ever realised that street extends to "EDGE" of COSMOS?

Would **you reach the speed of light,**
then you would — *at this very speed of light* —
instantly **smack against the "edge" of the universe!**

(*Whatever the latter might be*).

Special Relativity <u>presumes</u> ***symmetry***, both observers are equally equal.
This premise implies the evident but *implicit* assumption

that	***YOUR***	speed w.r.t. ***ME***,	as measured by ***ME***
equals	***MY***	speed w.r.t. ***YOU***,	as measured by ***YOU***.

$$v_{\mathbf{me}} = \Delta L_{\mathbf{me}}/\Delta t_{\mathbf{me}} = \Delta L_{\mathbf{you}}/\Delta t_{\mathbf{you}} = v_{\mathbf{you}}$$

(suffix means *observed by*, in hir own frame).

We usually take for granted

that	***YOUR***	speed w.r.t. ***ME***,	as measured by ***YOURSELF***
equals	***YOUR***	speed w.r.t. ***ME***,	as measured by ***ME***
AND	***MY***	speed w.r.t. ***YOU***,	as measured by ***MYSELF***
equals	***MY***	speed w.r.t. ***YOU***,	as measured by ***YOU***,

which inevitably renders *Lorentz contraction*:

due to time dilation, moving observer perceives shorter distance.

Would we however **reject** this **presumption *we merely took for granted***,
then both perceive the ***same length*** of the street, but the traveller needs less time as perceived by hirself, so ***(s)he*** perceives ***hir own*** velocity as way larger than how the ***stat. obs.*** perceives it. From hir own perspective, (s)he could go way faster than the *speed of light*! It is called ***celerity*** and it equals: $\Delta L_{\mathbf{stat}}/\Delta t_{\mathbf{mov}} = \gamma\beta c$.