Near the end of Einstein's very first publication of his ~~theory~~ conclusion of relativity, we find:

$$E_{\text{kin}} = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$$

Kinetic energy (*energy* of motion) as measured by a stationary observer equals a body's *mass* (measured when it is not moving) times the *speed of light* squared times the so-called *Lorentz factor* minus one. If you're not familiar with the latter, then please take this for granted, its explanation is irrelevant in the current context. "v" is the body's *velocity* and "c" is the *speed of light*. With the above, we can do some simple mathematics and since the *mass* was measured at rest, we'll give it a corresponding suffix and call it the ***rest mass***:

$$E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}} - m_{\text{rest}}c^2$$

is the same as: $$m_{\text{rest}}c^2 + E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}}$$

The actual meaning of $m_{\text{rest}}c^2$ seems still unclear, but it should be obvious that it must be ***some*** type of *energy*. We will call it the ***rest energy***:

$$\boldsymbol{E_{\text{rest}} = m_{\text{rest}}c^2}$$

which we incorporate in the ***total energy***:

$$E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}} = \frac{m_{\text{rest}}c^2}{\sqrt{1-v^2/c^2}}$$

Squaring this yields: $$E_{\text{tot}}^2 = \frac{m_{\text{rest}}^2c^4}{1-v^2/c^2} = \frac{E_{\text{rest}}^2}{1-v^2/c^2}$$

which we $$E_{\text{tot}}^2\left(1 - \frac{v^2}{c^2}\right) = E_{\text{rest}}^2$$

rewrite $$E_{\text{tot}}^2 - E_{\text{tot}}^2\frac{v^2}{c^2} = E_{\text{rest}}^2$$

step by $$E_{\text{tot}}^2 = E_{\text{rest}}^2 + E_{\text{tot}}^2\frac{v^2}{c^2}$$

step to: $$E_{\text{tot}}^2 = E_{\text{rest}}^2 + \frac{E_{\text{tot}}^2}{c^4}v^2c^2$$

Similar to $E_{\text{rest}} = m_{\text{rest}}c^2$, we can state: $\boldsymbol{E_{\text{tot}} = m_{\text{tot}}c^2}$ and we will call $\boldsymbol{m_{\text{tot}}}$ the ***total mass***.

therefore: $$\boldsymbol{E_{\text{tot}}^2 = m_{\text{rest}}^2c^4 + m_{\text{tot}}^2v^2c^2}$$

Together with: $$E_{\text{tot}}^2 = m_{\text{tot}}^2c^4$$

we obtain: $$m_{\text{tot}}^2c^4 = m_{\text{rest}}^2c^4 + m_{\text{tot}}^2v^2c^2$$

or: $$m_{\text{tot}}^2c^4 - m_{\text{tot}}^2v^2c^2 = m_{\text{rest}}^2c^4$$

Division by c^4: $$m_{\text{tot}}^2 - m_{\text{tot}}^2v^2/c^2 = m_{\text{rest}}^2$$

renders: $$m_{\text{tot}}^2(1 - v^2/c^2) = m_{\text{rest}}^2$$

hence: $$\boldsymbol{m_{\text{tot}}^2 = m_{\text{rest}}^2/(1 - v^2/c^2)}$$

Similar to: $$E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}}$$

we can write: $$m_{\text{tot}} = m_{\text{rest}} + m_{\text{kin}}$$

We already have: $$E_{\text{rest}} = m_{\text{rest}}c^2$$

as well as: $$E_{\text{tot}} = m_{\text{tot}}c^2$$

so it must be that: $$E_{\text{kin}} = m_{\text{kin}}c^2$$

altogether, we find: $$\boldsymbol{m_{\text{tot}} = m_{\text{rest}} + m_{\text{kin}} = m_{\text{rest}}/\sqrt{1 - v^2/c^2}}$$

Multiplied by c^2: $$\boldsymbol{E_{\text{tot}} = E_{\text{rest}} + E_{\text{kin}} = E_{\text{rest}}/\sqrt{1 - v^2/c^2}}$$

Adding *energy* behaves like adding *mass*! In this context, *mass* should be seen as *inertia*, the resistence against change of motion. *Energy* appears to have *inertia* too! However, this is only observed by a stationary observer. The moving body does not experience this in its own frame of reference.

In the found: $$E_{tot}^2 = m_{rest}^2 c^4 + m_{tot}^2 v^2 c^2$$

we can substitute: $\boldsymbol{m_{tot} v} = m_{rest} v/\sqrt{1 - v^2/c^2} = p_{class}/\sqrt{1 - v^2/c^2} = \boldsymbol{p_{tot}}$

where p_{class} is the *momentum* as defined in classical mechanics. Then p_{tot} is of course the ***total momentum*** of the moving body, which in practice is called the ***relativistic momentum*** (the *total mass* is actually called *relativistic* as well).

We then obtain: $$\boldsymbol{E_{tot}^2 = m_{rest}^2 c^4 + p_{tot}^2 c^2} = E_{rest}^2 + p_{tot}^2 c^2$$

People omitting these suffixes, forming a class of clever clearheaded but clearly clumsy claustrophobic clerical clowns in clubrooms of classical cloisters, claiming cloudy clarifications of the wrongness of:

$\boldsymbol{E = mc^2}$ (*see further below*)

because it should be: $E^2 = (mc^2)^2 + (pc)^2$ not only compare, but even equate apples to oranges.

If we assign a *mass* to a photon: $E_\gamma = m_\gamma c^2 \therefore m_\gamma = E_\gamma / c^2$

then its *rest mass*: $m_{rest,\gamma} = m_\gamma \sqrt{1 - \frac{(v_\gamma = c)^2}{c^2}} = 0$ (although a photon can't be at rest)

equals nought, hence: $E_{tot}^2 = p_{tot}^2 c^2$

After replacing the suffix, we find: $E_\gamma = p_\gamma c$

Einstein got his Nobel Prize for: $E_\gamma = h\nu$ (please note: "ν" is the greek letter *nu*)

so **a photon's *momentum*** equals: $\boldsymbol{p_\gamma = h\nu / c}$

We have found: $E_{rest/tot/kin} = m_{rest/tot/kin} c^2$

which in general becomes: $$\boldsymbol{E = mc^2}$$

This is often called the most famous equation of all times. In his very next publication, Einstein demonstrates that a radiating body must lose *mass* according to this equation. It suddenly explained how the sun is able to shine so luminously for billions of years! In its interior, the sun actually converts *mass* to *energy*. It is also what drives nuclear power plants and unfortunately, we must add atomic bombs to this list too. Annihilation reactions of elementary particles and their antiparticles show that the entire *mass* of a particle can be converted to *energy* according to Einstein's formula. *Mass* is in fact a condensed form of *energy*.

The speed of light is: $c = 299\,792\,458$ m/s ≈ 1 billion (Dutch: 1 miljard) km/h. When we square that, we obtain $89\,875\,517\,873\,681\,764 \approx 10^{17}$ m²/s². This means that even a very small *mass* could be converted to a very **HUGE** amount of *energy*!

Example 1: The sun continuously emits $3.828 \times 10^{26} = 382\,800\,000\,000\,000\,000\,000\,000\,000$ watts of power in all directions. That corresponds to $1.344 \times 10^{17} = 134\,400\,000\,000\,000\,000$ kilograms per year or 1 earth mass in 44.4 million years, 225 earth masses in 10 billion (Dutch: 10 miljard) years. The total mass of the sun is 333 000 times that of the earth, so the sun will "burn" merely $0.00068 \approx 2/3$ ‰ of its total mass during its entire lifetime.

Example 2: In (actually above) Hiroshima, 0.75 g was converted to *energy* and in Nagasaki it was 1 g. This 1 gram yielded 90 million megajoules of energy. An electric car (as used on average in The Netherlands) consumes roughly 2.5 MJ/km, so this single gram would suffice for 36 million kilometres, which is roughly 100 times the distance to the moon or one quarter of the distance to the sun.

Brook Taylor found in 1715 that: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$

If x is small we can say: $\frac{1}{\sqrt{1-x^2}} = 1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{3x^6}{16} + \mathcal{O}(x^8)$

hence: $\boldsymbol{E_{kin}} = mc^2 \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) \approx mc^2 \left(1 + \frac{v^2/c^2}{2} - 1 \right) = \boldsymbol{\frac{1}{2} m v^2}$

which is the classical equation for *kinetic energy*. It applies if $v \ll c$, i.e. in daily life.

